

Kirchhoff's Laws

Kirchhoff's Current Law

- Or KCL for short
 - Based upon conservation of charge – the algebraic sum of the charge within a system can not change.

$$\sum_{n=1}^N i_n = 0$$

Where N is the total number of branches connected to a node.

$$\sum_{\text{node}} i_{\text{enter}} = \sum_{\text{node}} i_{\text{leave}}$$

Kirchhoff's Voltage Law

- Or KVL for short
 - Based upon conservation of energy – the algebraic sum of voltages dropped across components around a loop is zero.

$$\sum_{m=1}^M v = 0$$

Where M is the total number of branches in the loop.

$$\sum v_{\text{drops}} = \sum v_{\text{rises}}$$

Example 1

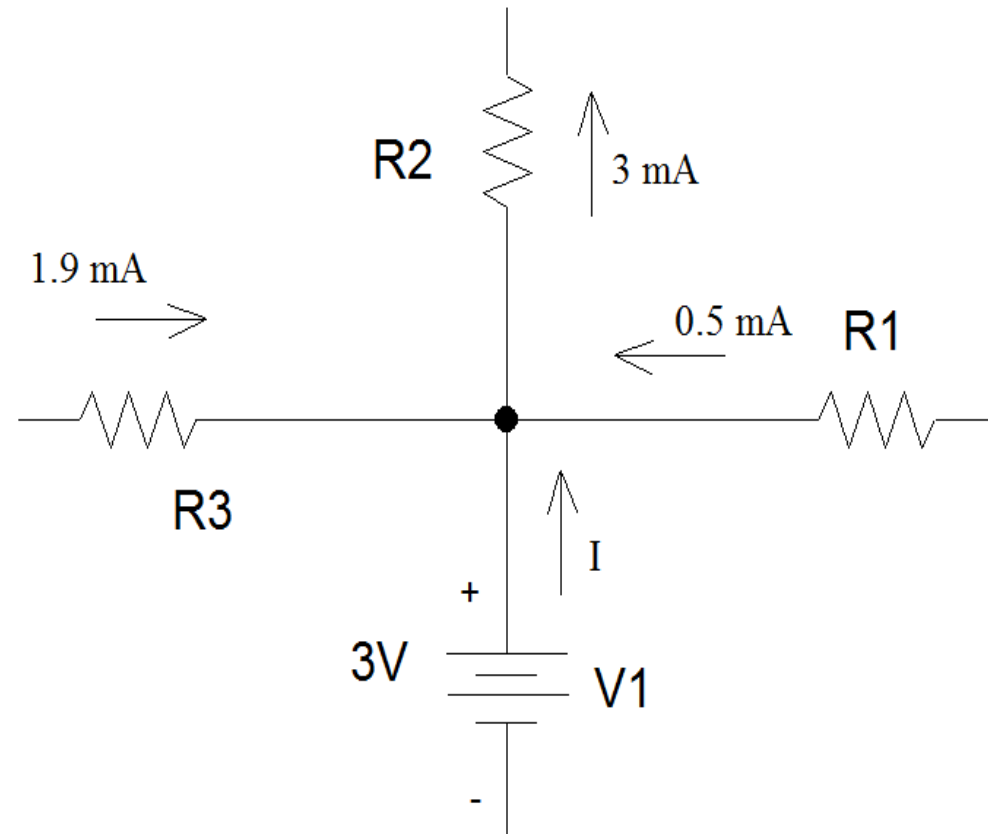
- Determine I , the current flowing out of the voltage source.
 - Use KCL
 - $1.9\text{ mA} + 0.5\text{ mA} + I$ are entering the node.
 - 3 mA is leaving the node.

$$1.9\text{ mA} + 0.5\text{ mA} + I = 3\text{ mA}$$

V_1 is generating power.

$$I = 3\text{ mA} - (1.9\text{ mA} + 0.5\text{ mA})$$

$$I = 0.6\text{ mA}$$



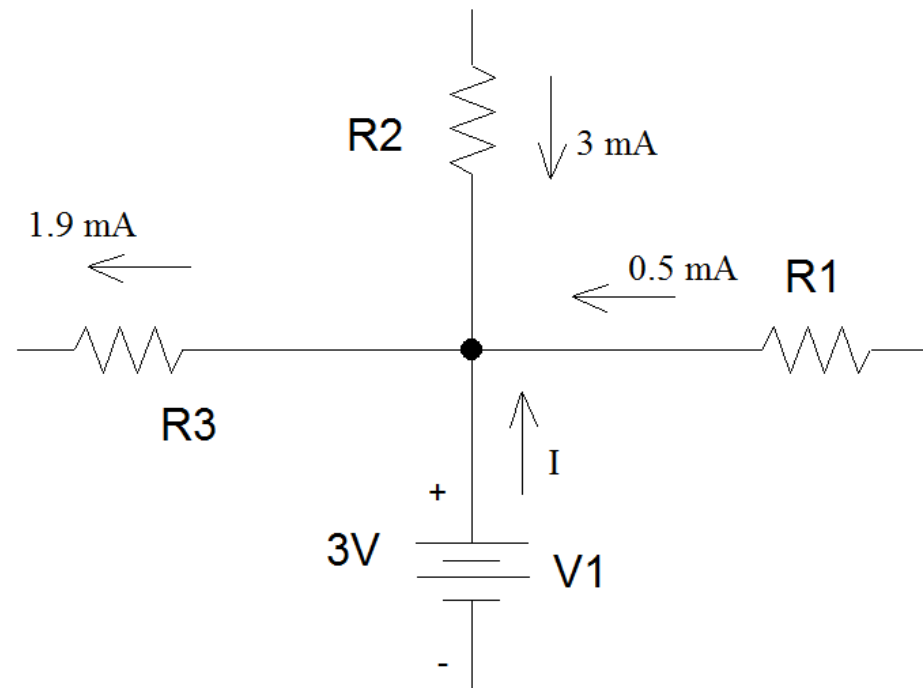
Example 2

- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.
 - Use KCL
 - $3\text{ mA} + 0.5\text{ mA} + I$ are entering the node.
 - 1.9 mA is leaving the node.

$$3\text{mA} + 0.5\text{mA} + I = 1.9\text{mA}$$

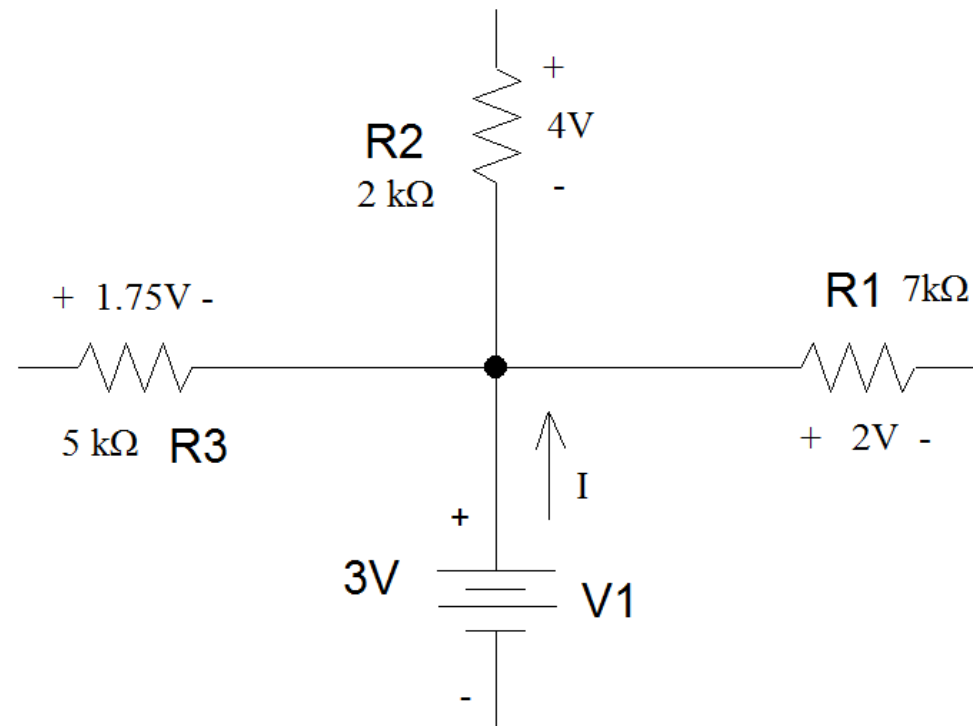
$$I = 1.9\text{mA} - (3\text{mA} + 0.5\text{mA})$$

$$I = -1.6\text{mA}$$



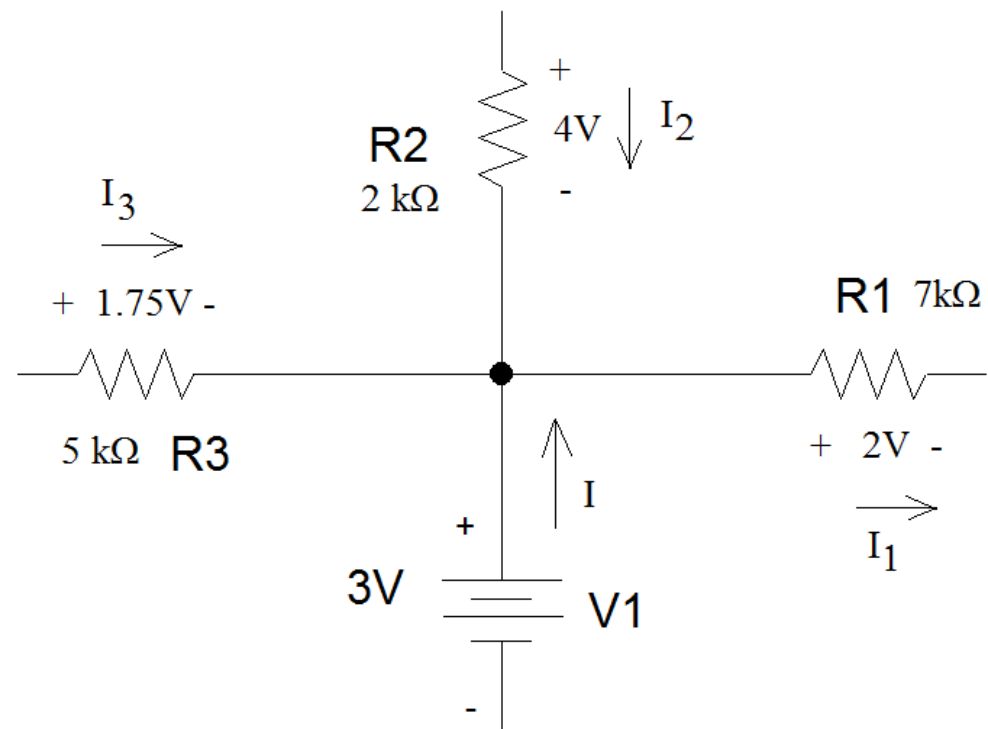
Example 3

- If voltage drops are given instead of currents, you need to apply Ohm's Law to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.



Example 3 (con't)

- For power dissipating components such as resistors, passive sign convention means that current flows into the resistor at the terminal has the + sign on the voltage drop and leaves out the terminal that has the – sign.

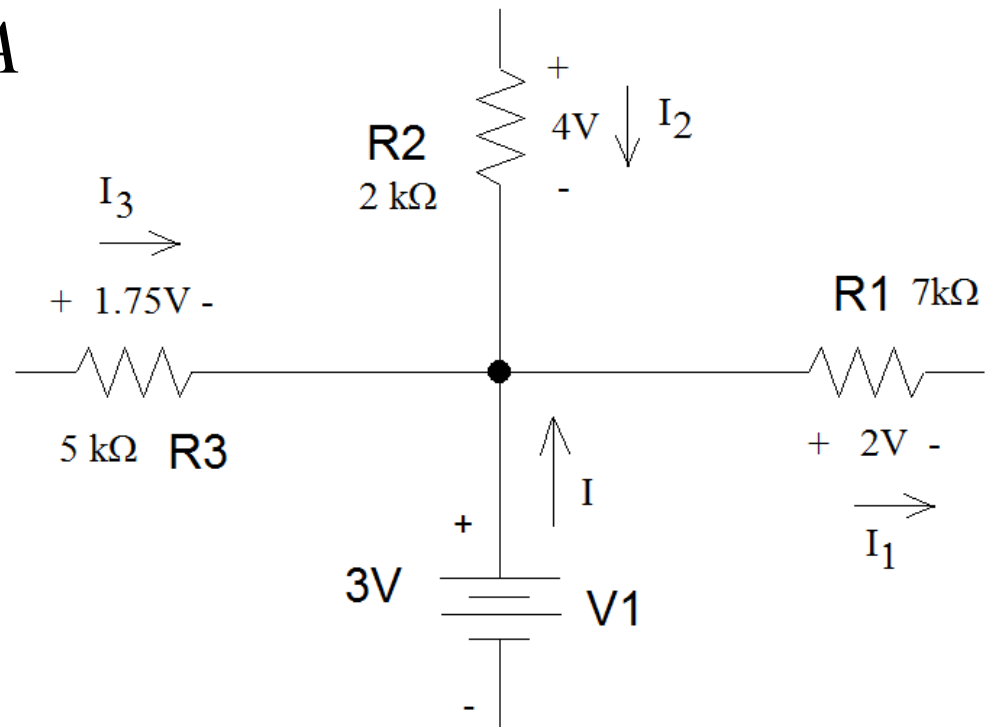


Example 3 (con't)

$$I_1 = 2V / 7k\Omega = 0.286mA$$

$$I_2 = 4V / 2k\Omega = 2mA$$

$$I_3 = 1.75V / 5k\Omega = 0.35mA$$



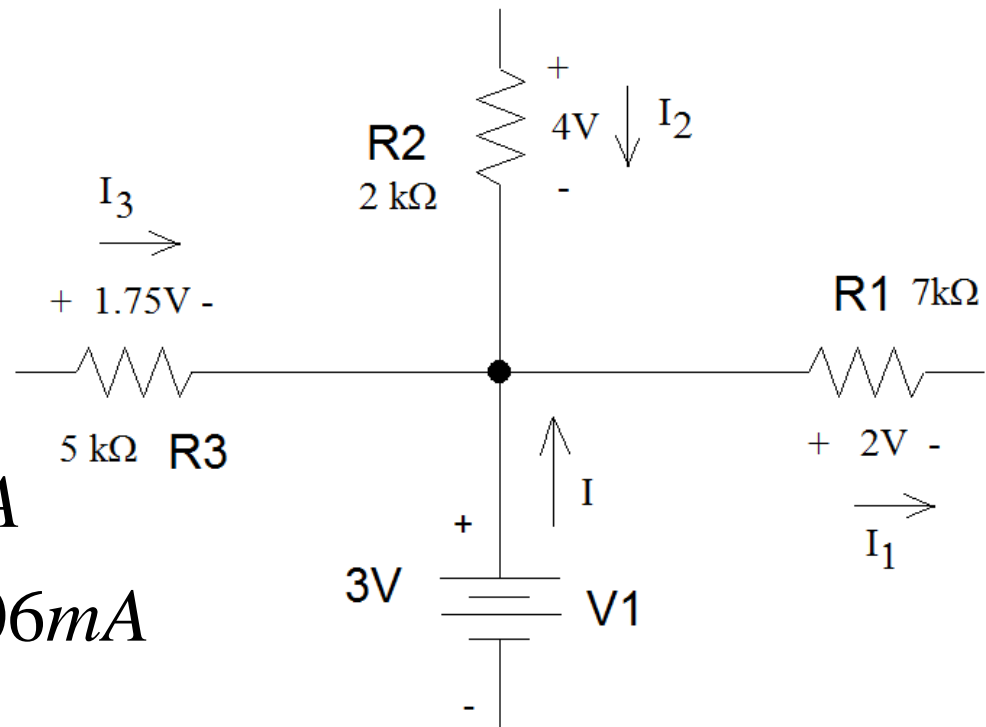
Example 3 (con't)

- I_1 is leaving the node.
- I_2 is entering the node.
- I_3 is entering the node.
- I is entering the node.

$$I_2 + I_3 + I = I_1$$

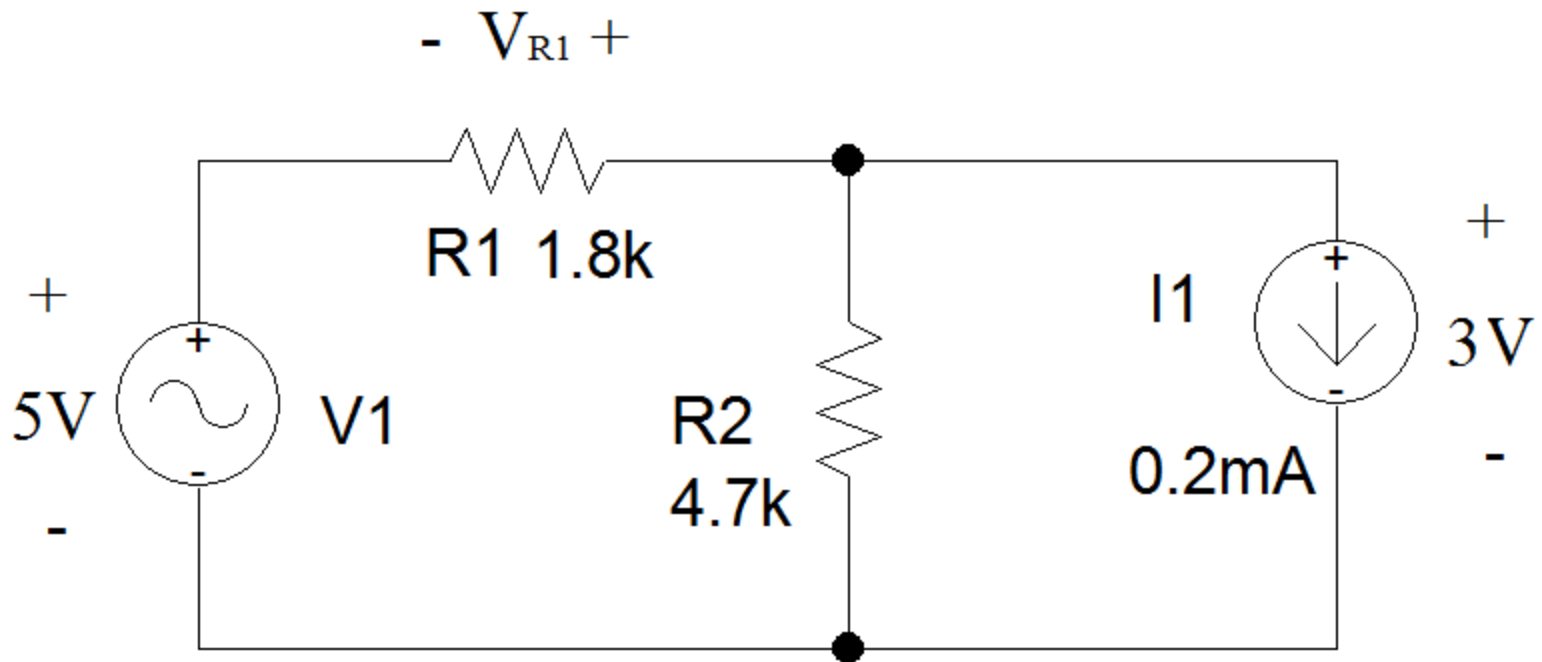
$$2mA + 0.35mA + I = 0.286mA$$

$$I = 0.286mA - 2.35mA = -2.06mA$$



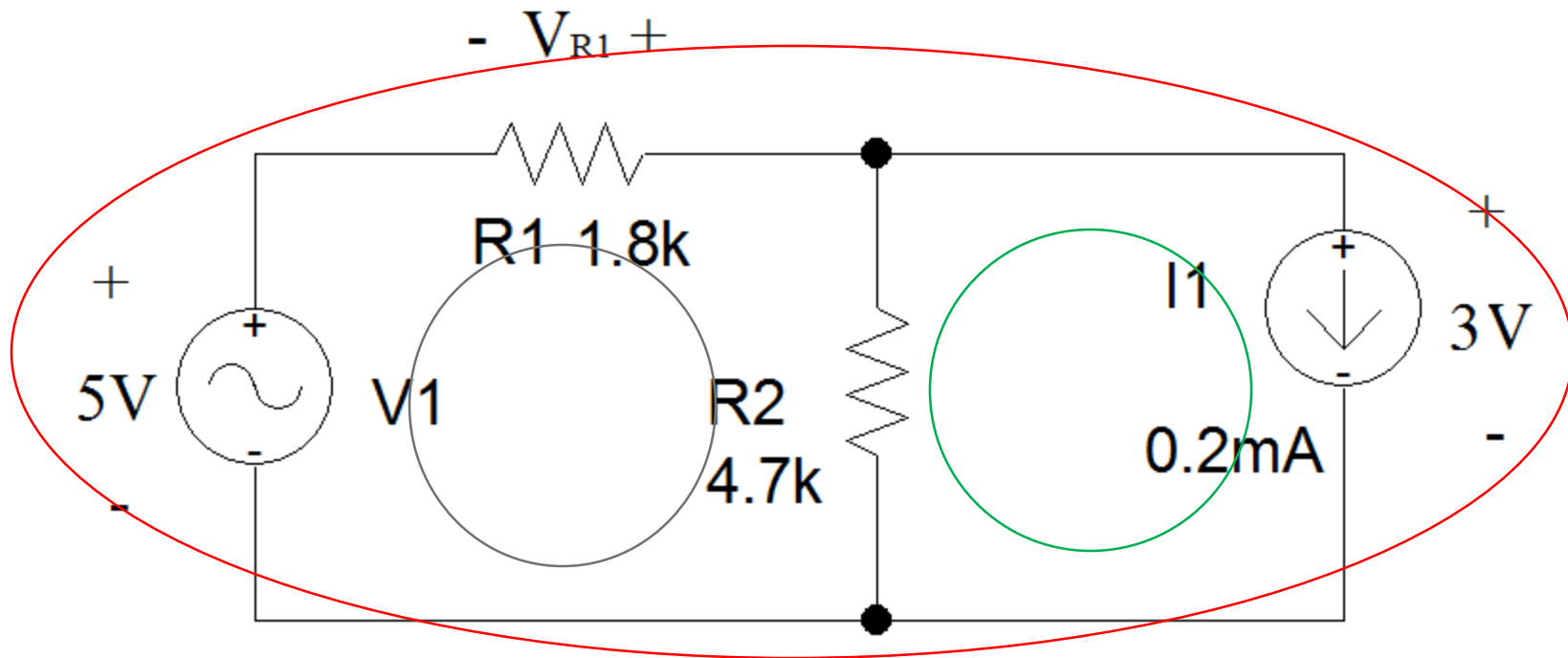
Example 4

- Find the voltage across R1. Note that the polarity of the voltage has been assigned in the circuit schematic.
 - First, define a loop that include R1.



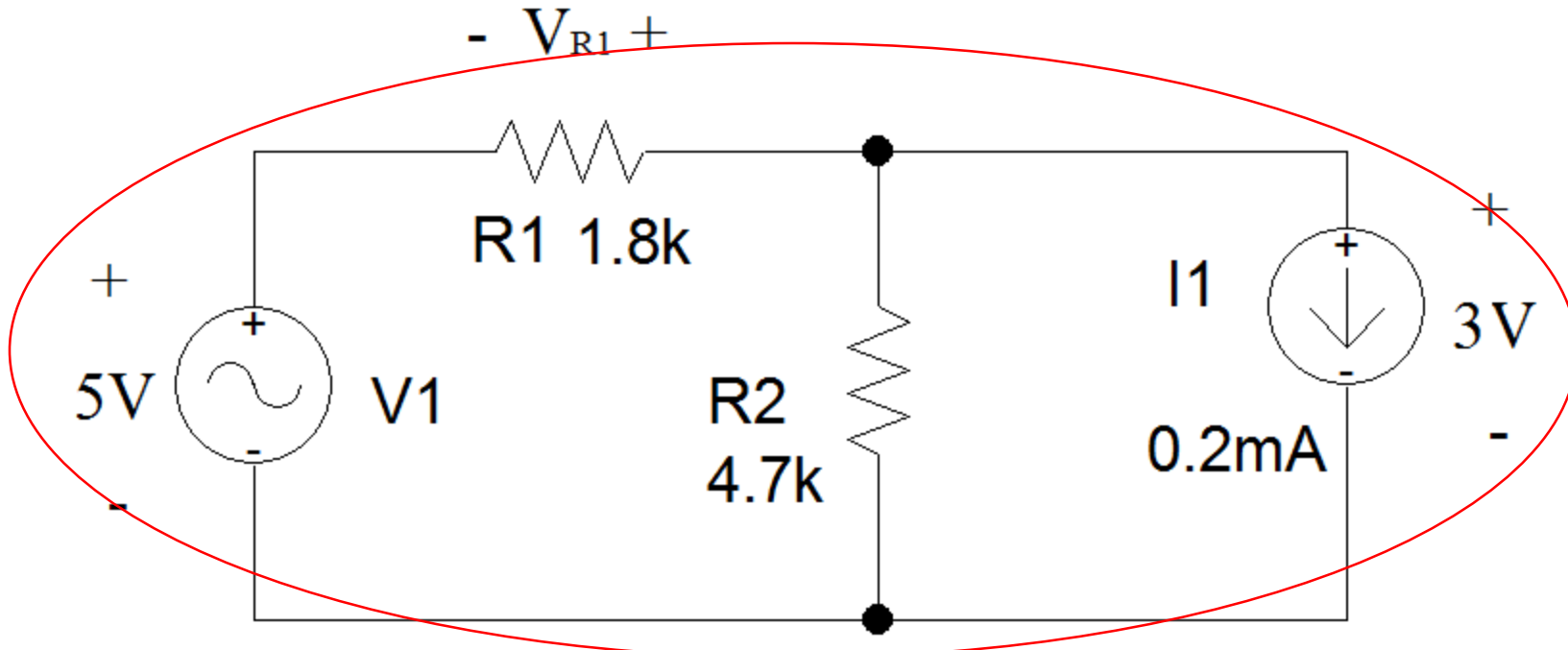
Example 4 (con't)

- There are three possible loops in this circuit – only two include R1.
 - Either loop may be used to determine V_{R1} .



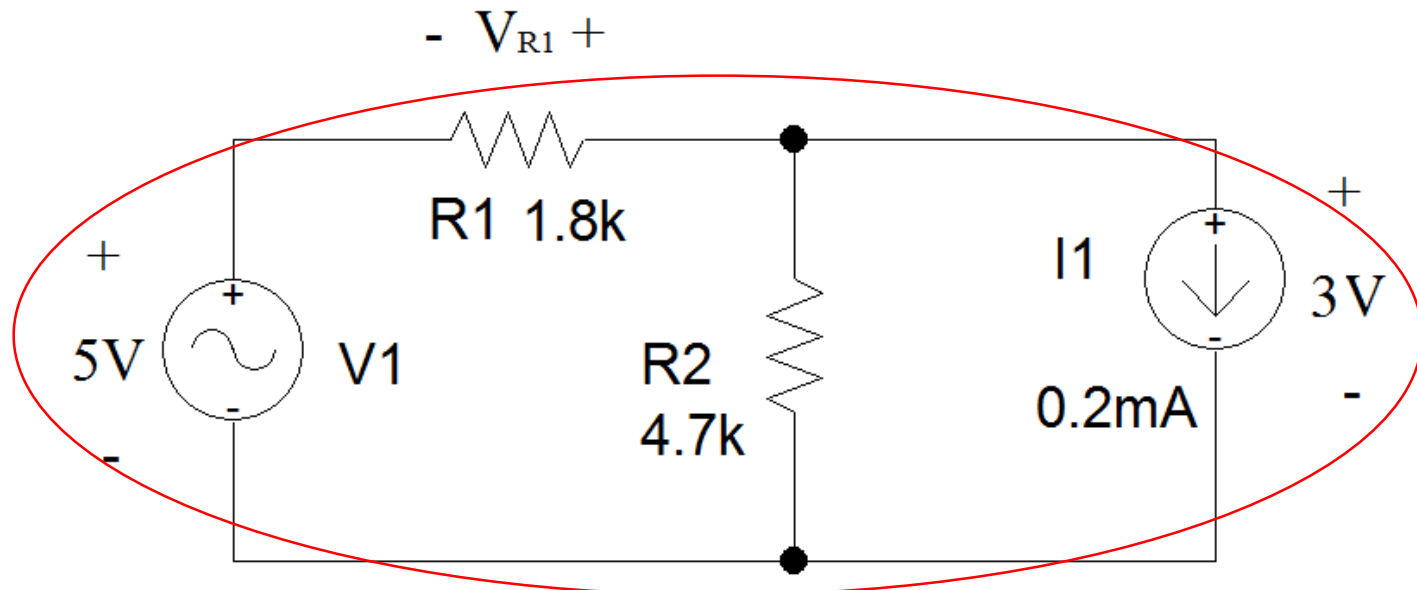
Example 4 (con't)

- If the outer loop is used:
 - Follow the loop clockwise.



Example 4 (con't)

- Follow the loop in a clockwise direction.
- The 5V drop across V1 is a voltage rise.
- V_{R1} should be treated as a voltage rise.
- The loop enters R2 on the positive side of the voltage drop and exits out the negative side. This is a voltage drop as the voltage becomes less positive as you move through the component.

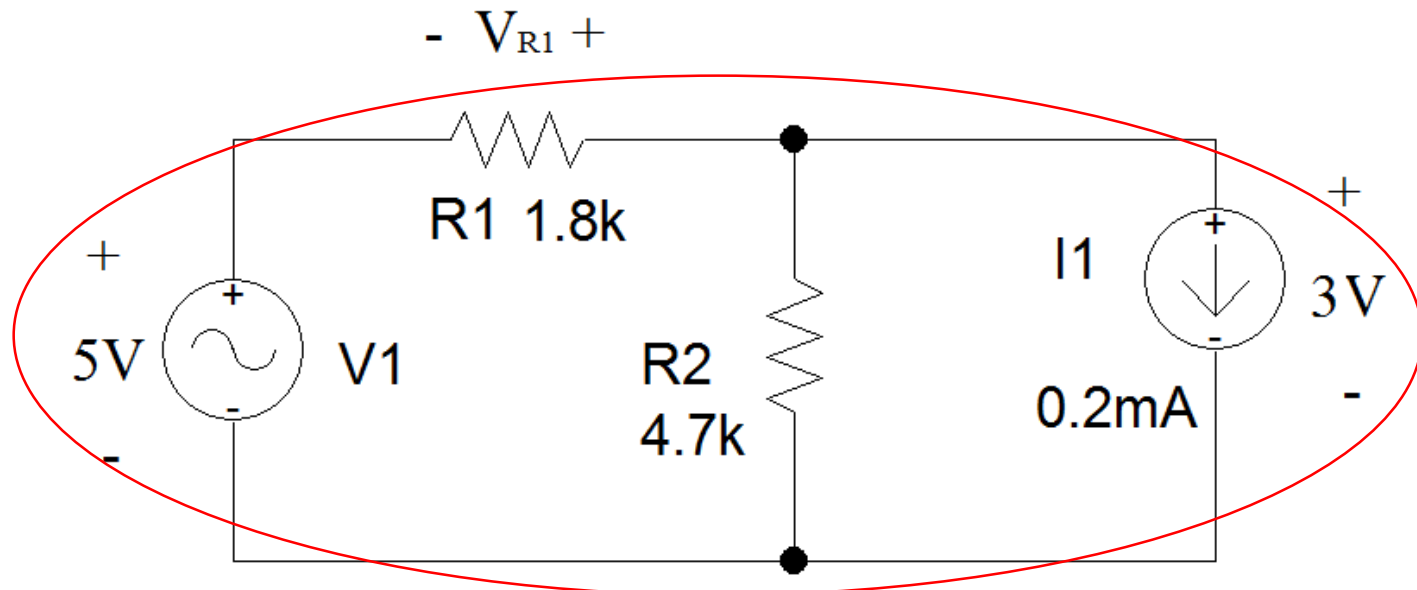


Example 4 (con't)

- By convention, voltage drops are added and voltage rises are subtracted in KVL.

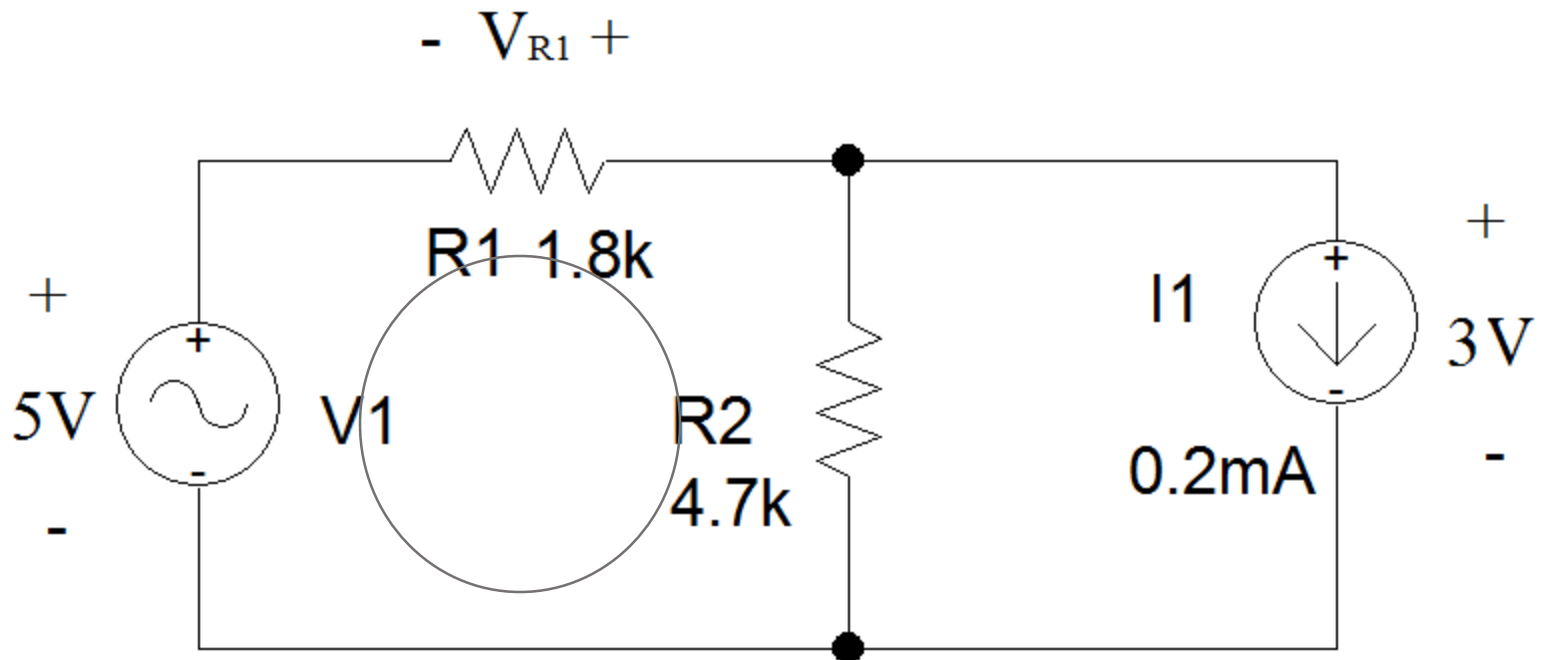
$$-5V - V_{R1} + 3V = 0$$

$$V_{R1} = -2V$$



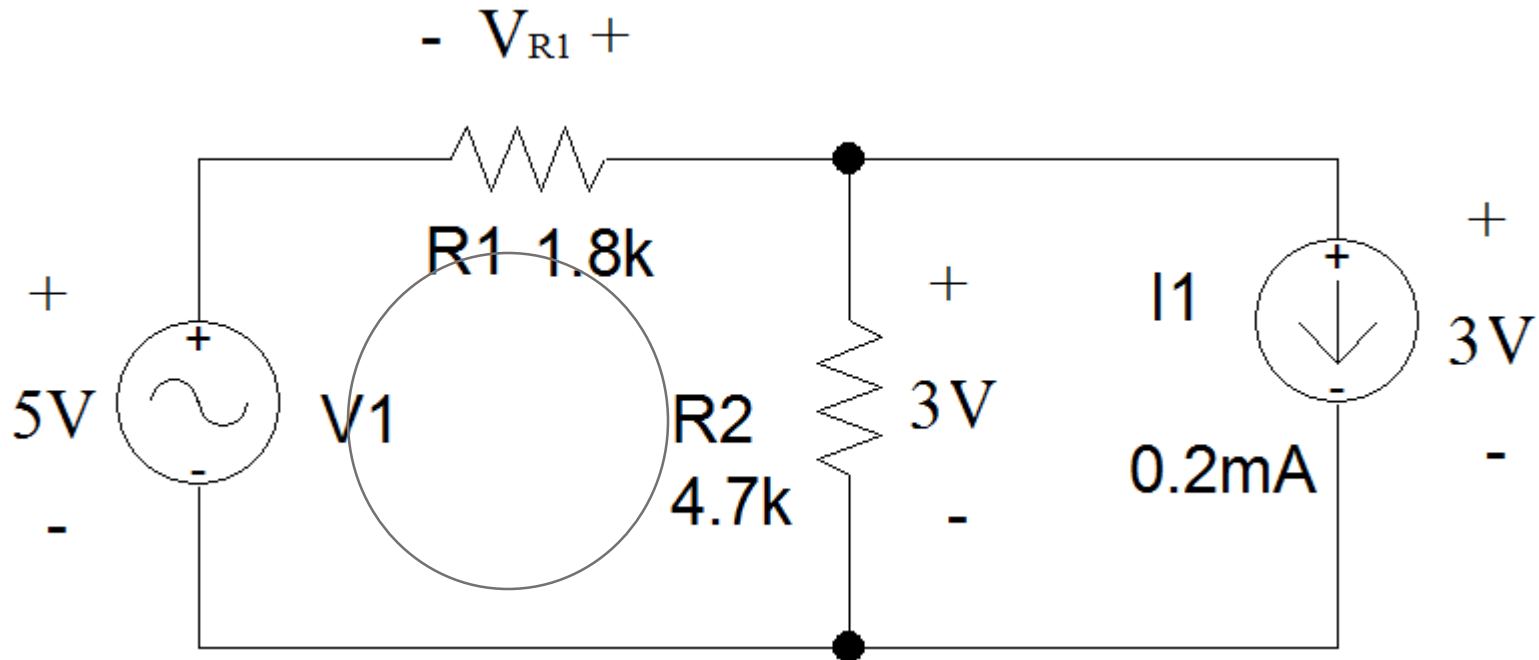
Example 4 (con't)

- Suppose you chose the blue loop instead.
 - Since R2 is in parallel with I1, the voltage drop across R2 is also 3V.



Example 4 (con't)

- The 5V drop across V1 is a voltage rise.
- V_{R1} should be treated as a voltage rise.
- The loop enters R2 on the positive side of the voltage drop and exits out the negative side. This is a voltage drop as the voltage becomes less positive as you move through the component.

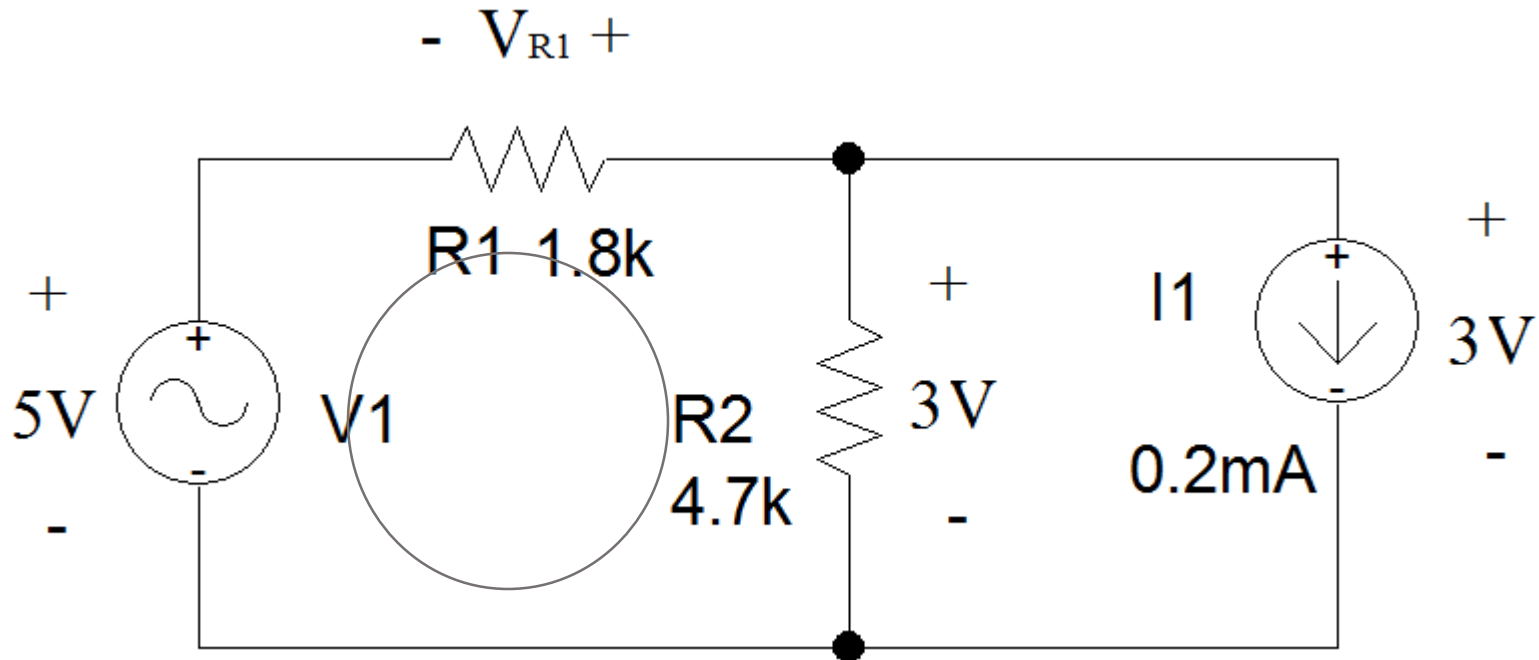


Example 4 (con't)

- As should happen, the answer is the same.

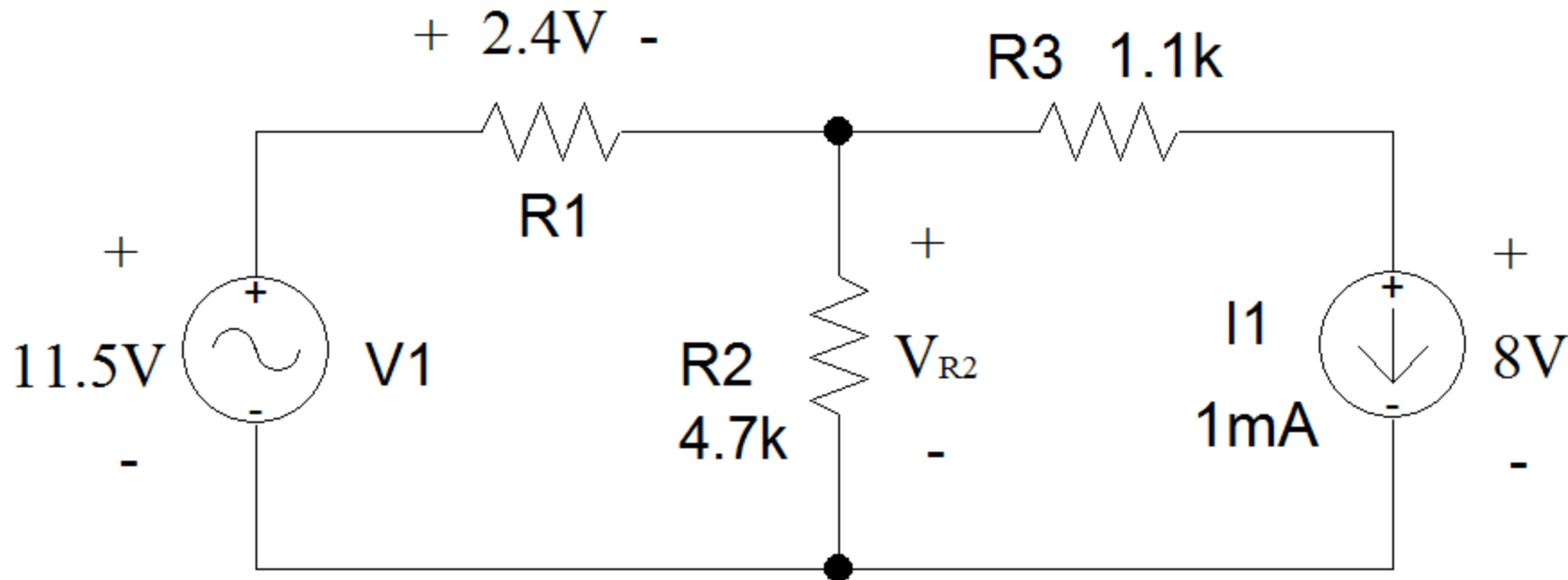
$$-5V - V_{R1} + 3V = 0$$

$$V_{R1} = -2V$$



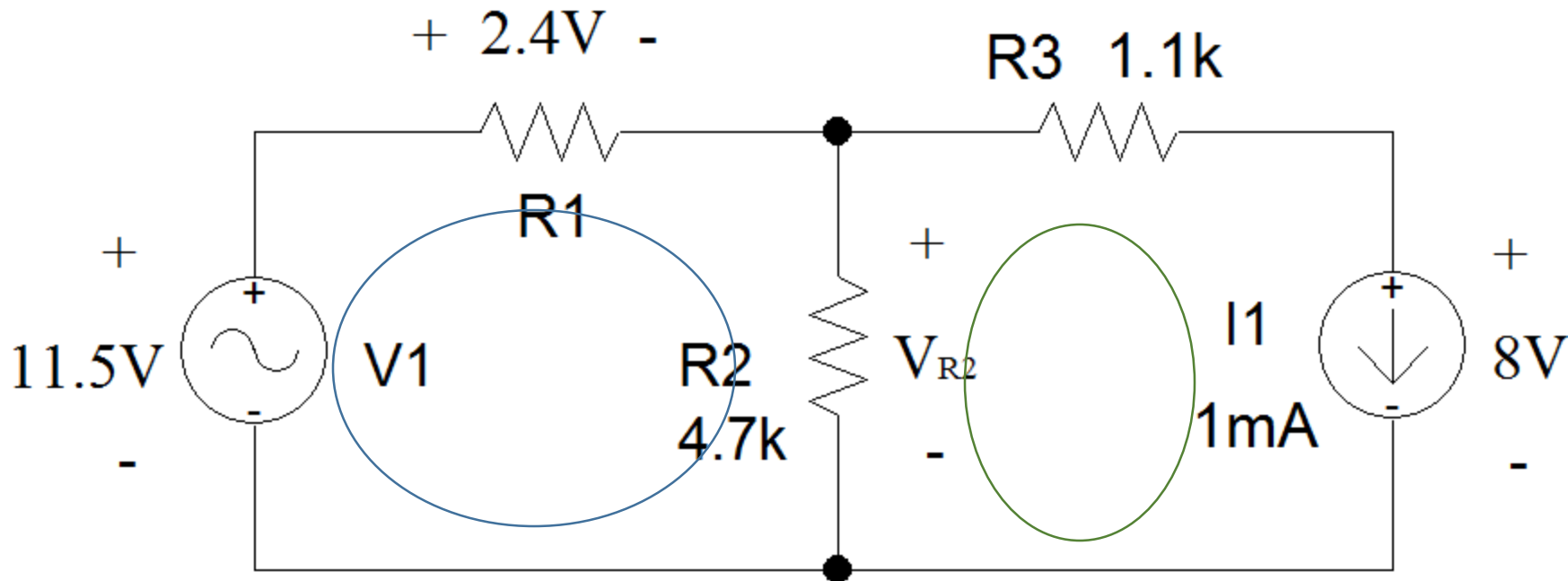
Example 5

- Find the voltage across R2 and the current flowing through it.
 - First, draw a loop that includes R2.



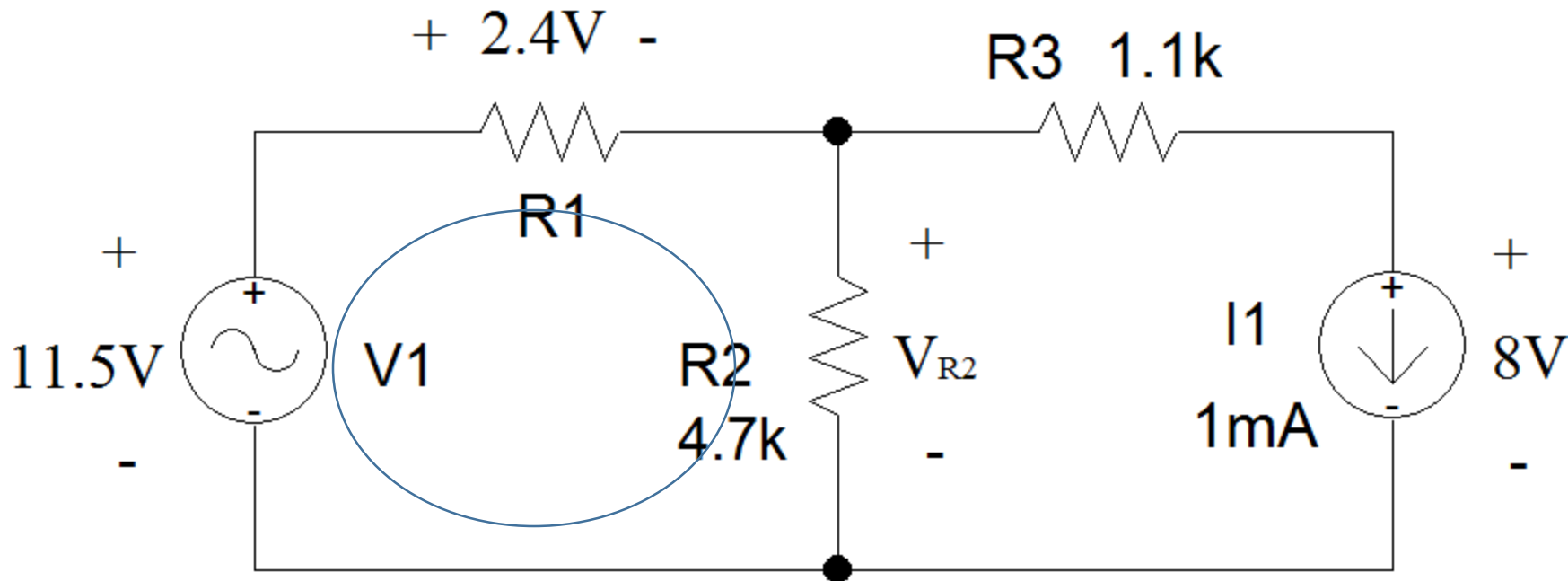
Example 5 (con't)

- There are two loops that include R2.
 - The one on the left can be used to solve for V_{R2} immediately.



Example 5 (con't)

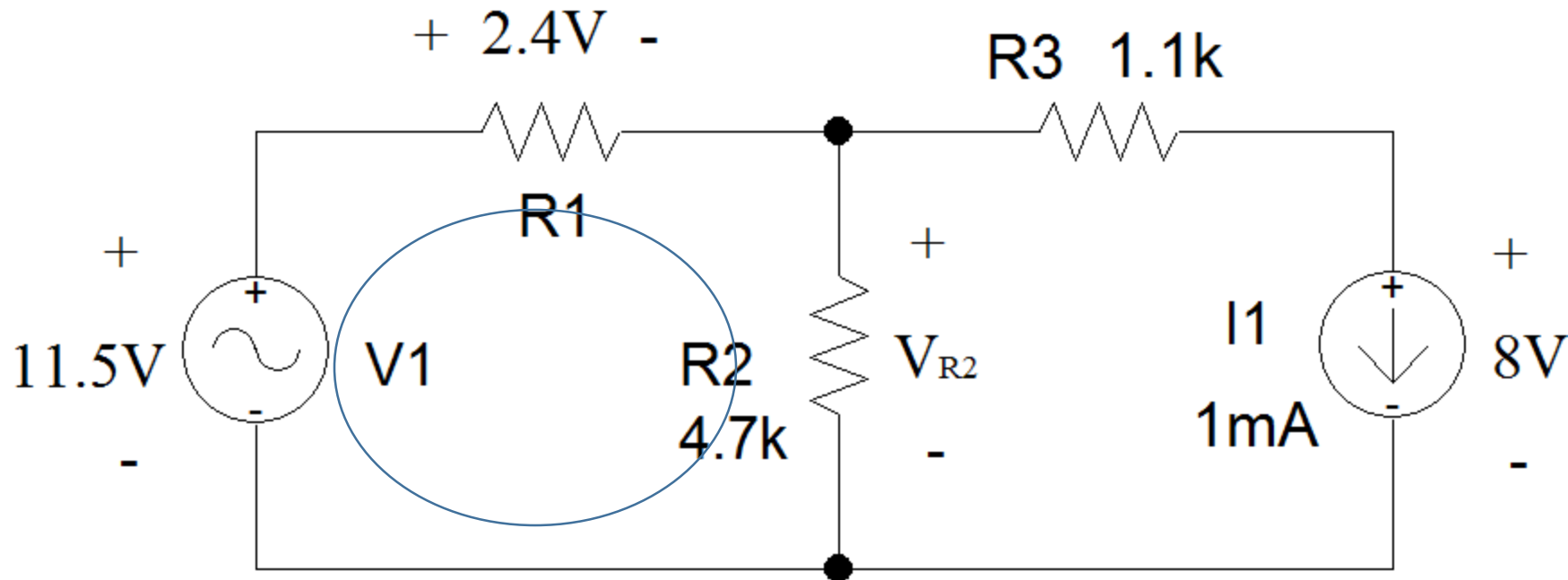
- Following the loop in a clockwise direction.
 - The 11.5V drop associated with V1 is a voltage rise.
 - The 2.4V associated with R1 is a voltage drop.
 - V_{R2} is treated as a voltage drop.



Example 5 (con't)

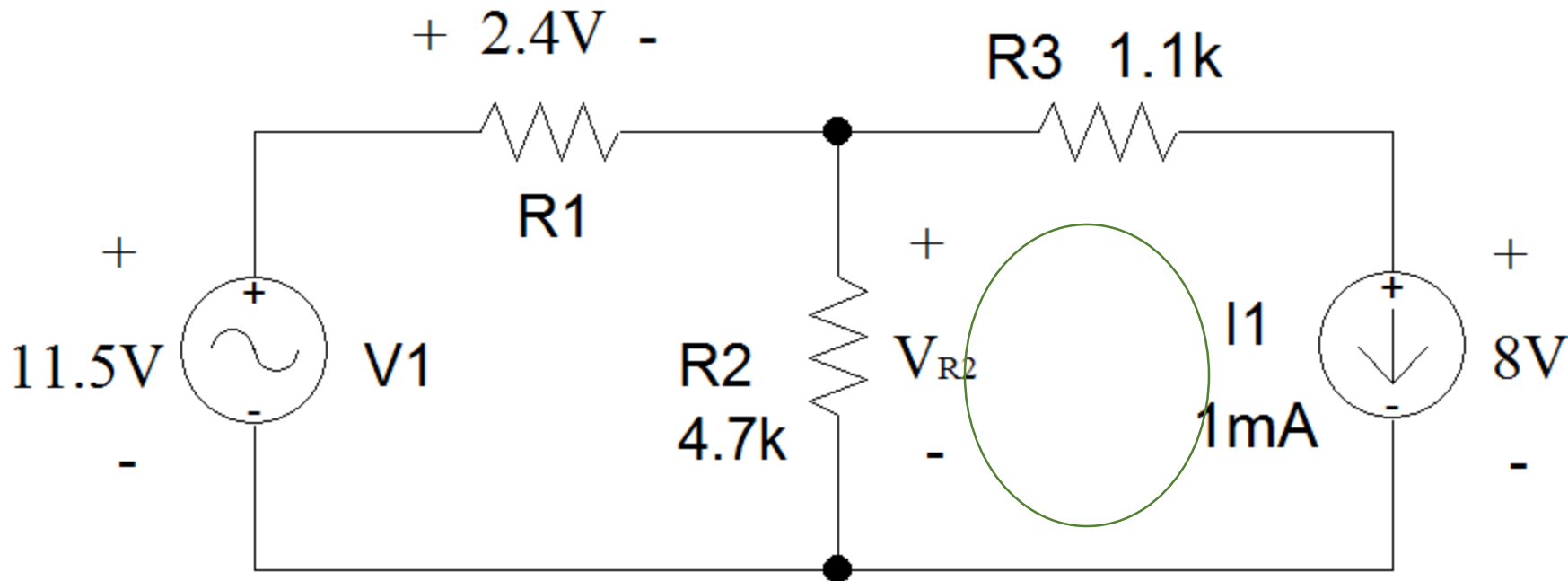
$$-11.5V + 2.4V + V_{R2} = 0$$

$$V_{R2} = 9.1V$$



Example 5 (con't)

- If you used the right-hand loop, the voltage drop across R3 must be calculated using Ohm's Law.



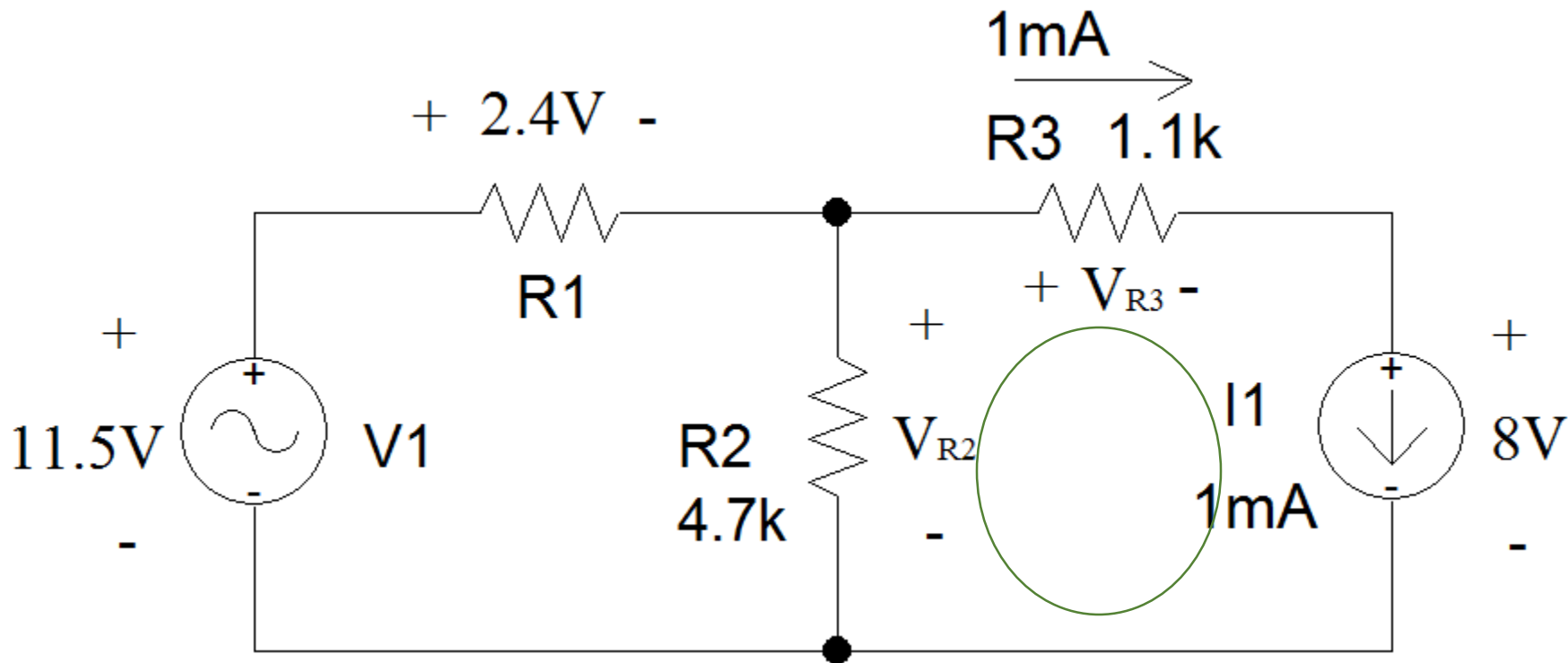
Example 5 (con't)

- Since R_3 is a resistor, passive convention means that the positive sign of the voltage drop will be assigned to the end of R_3 where current enters the resistor.
- As I_1 is in series with R_3 , the direction of current through R_3 is determined by the direction of current flowing out of the current source.
- Because I_1 and R_3 are in series, the magnitude of the current flowing out of I_1 must be equal to the magnitude of the current flowing out of R_3 .

Example 5 (con't)

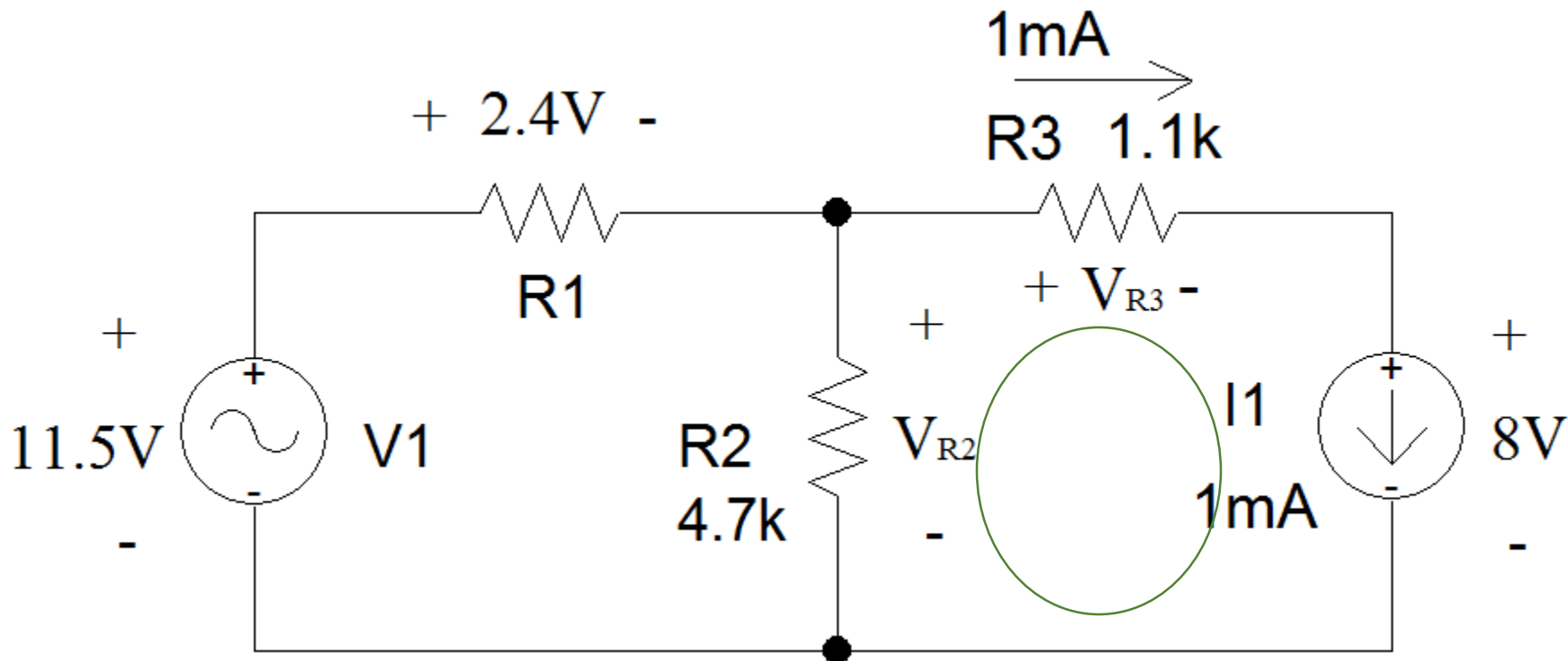
- Use Ohm's Law to find V_{R3} .

$$V_{R3} = 1mA(1.1k\Omega) = 1.1V$$



Example 5 (con't)

- Moving clockwise around the loop:
 - V_{R3} is a voltage drop.
 - The voltage associated with $I1$ is a voltage drop.
 - V_{R2} is a voltage rise.

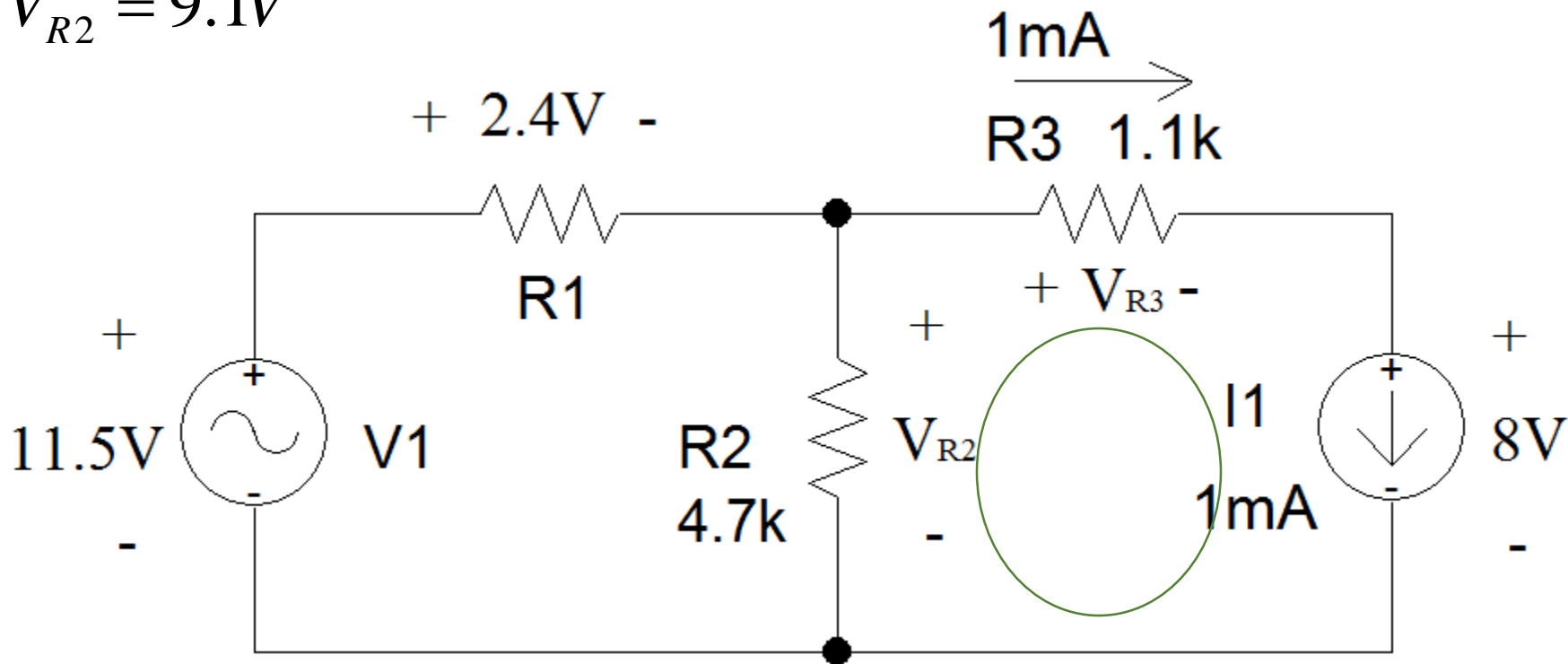


Example 5 (con't)

- Again, the same answer is found.

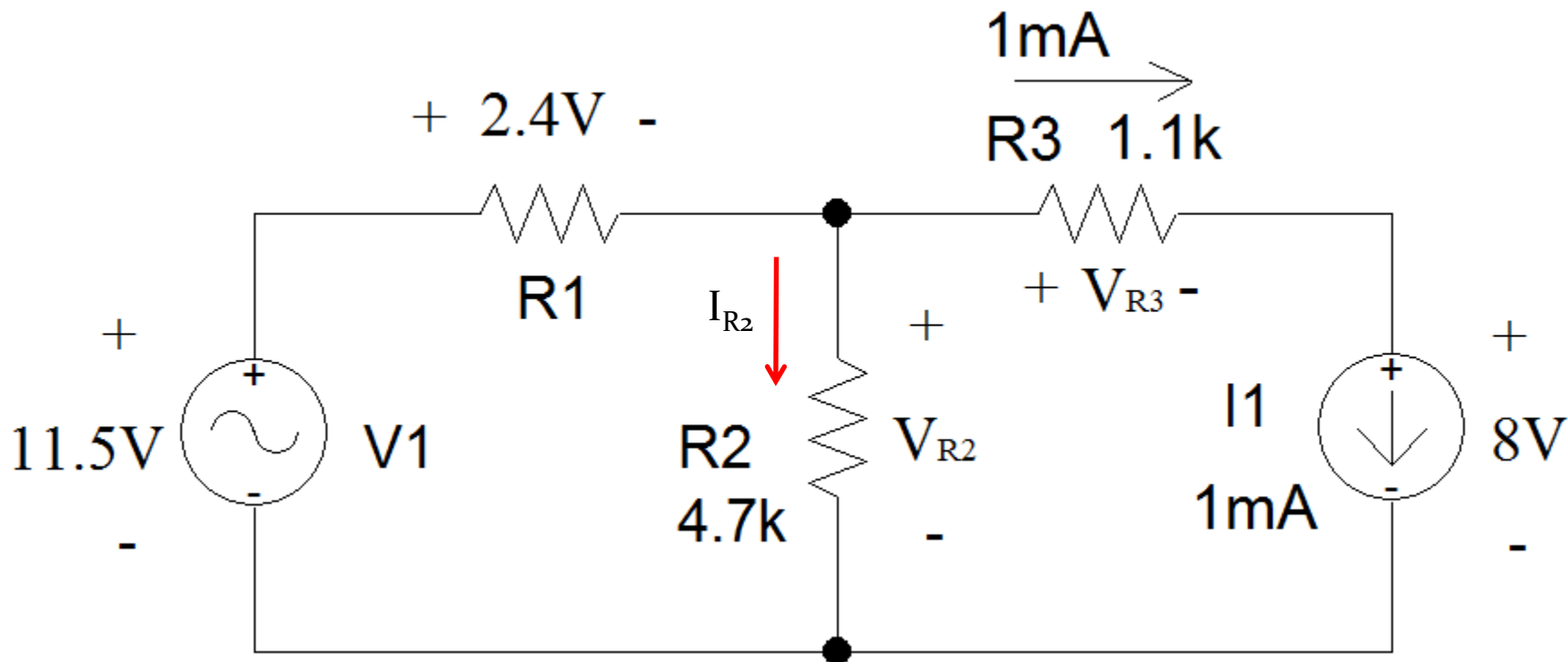
$$1.1V + 8V - V_{R2} = 0$$

$$V_{R2} = 9.1V$$



Example 5 (con't)

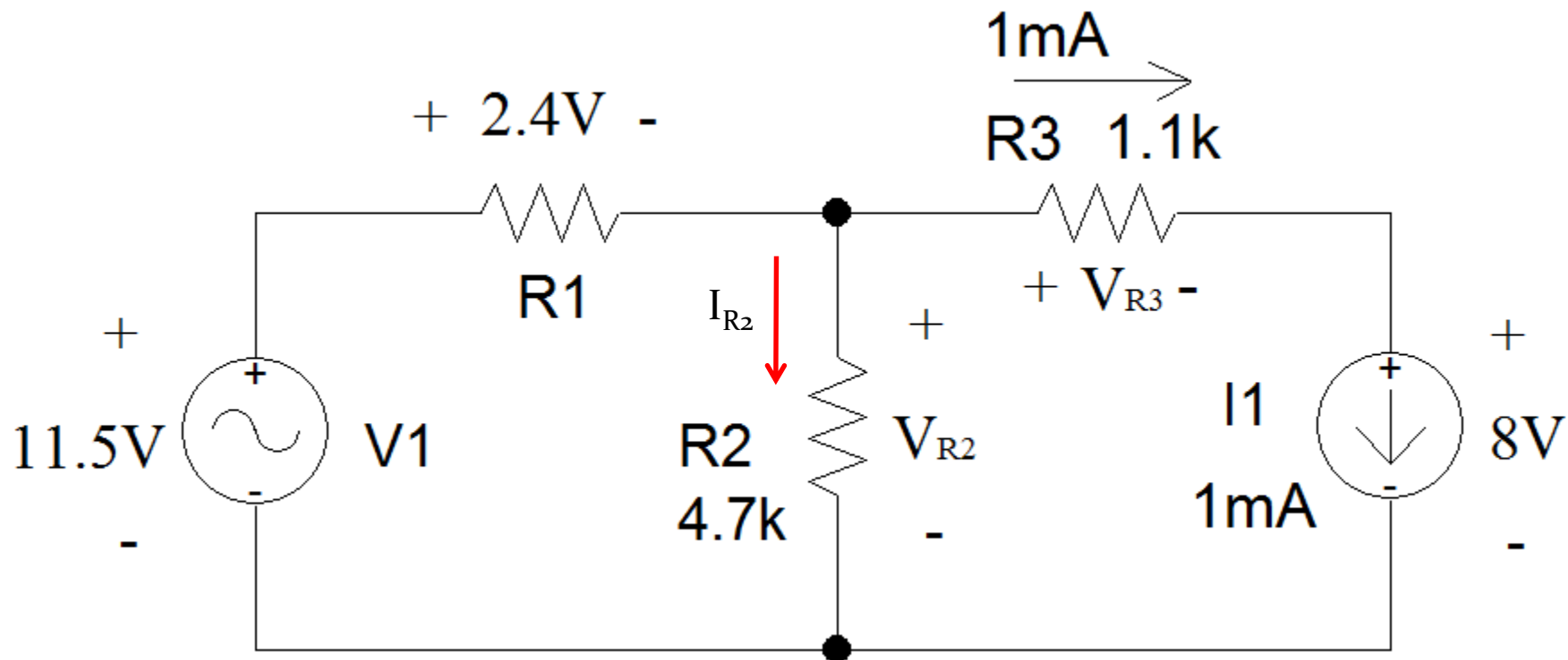
- Once the voltage across R2 is known, Ohm's Law is applied to determine the current.
 - The direction of positive current flow, based upon passive sign convention is shown in red.



Example 5 (con't)

$$I_{R2} = 9.1V / 4.7k\Omega$$

$$I_{R2} = 1.94mA$$



Note:

- If you use KCL and Ohm's Law, you could find out what the value of R_1 is in Example 5.

Summary

- The currents at a node can be calculated using Kirchhoff's Current Law (KCL).
- The voltage dropped across components can be calculated using Kirchhoff's Voltage Law (KVL).
- Ohm's Law is used to find some of the needed currents and voltages to solve the problems.